TMS Pr. 10.7

DETERMINE HOW MUCH GREATER THE GRAVITATIONAL FIELD STRENGTH IS AT THE POLE THAN AT THE EQUATOR. ASSUME A SPHERICAL EARTH. IF THE ACTUAL DIFFERENCE IS $\Delta g = 52\,mm/s^2$, EXPLAIN THE DIFFERENCE. LOOK UP OTHER DATA TO SEE IF YOU CAN CALCULATE A CLOSER VALUE.

$\vec{g}$ IS ACTUALLY A SUM OF THE GRAVITATIONAL & CENTRIFUGAL FORCES.

$$mg = \frac{GMm}{R_E^2} - m(\vec{\omega} \times (\vec{\omega} \times \vec{R}_E))$$

- AT THE POLE, $\vec{R}_E$ AND $\vec{\omega}$ ARE PARALLEL! ($R_E = 6371\,km$)
  $\Rightarrow$ THE CROSS PRODUCTS ARE ZERO!

$$\cancel{m}g_{POLE} = \frac{GM\cancel{m}}{R_E^2} \qquad \left(\frac{GM}{R_p^2} = 9.864\,\frac{m}{s^2}\right)$$

- AT THE EQUATOR, $\vec{R}_E$ IS $\perp$ TO $\vec{\omega}$
  $\Rightarrow \vec{\omega} \times (\vec{\omega} \times \vec{R}) = \omega^2 R$

$$\cancel{m}g_{EQ} = \frac{GM\cancel{m}}{R_E^2} - \cancel{m}\omega^2 R_E \qquad \left(\frac{GM}{R_E^2} = 9.799\,\frac{m}{s^2}\right)$$

$$g_{EQ} = 9.765\,\frac{m}{s^2}$$

THE DIFFERENCE IS

$$\Delta g = g_{POLE} - g_{EQ} = \omega^2 R_E$$

$$\text{FOR } \omega = \frac{2\pi}{T_E} = \frac{2\pi}{(24)(3600)} = 7.27 \times 10^{-5}\,\frac{RAD}{s}$$

$$\Delta g = (7.27 \times 10^{-5})^2 (6371 \times 10^3) = 0.0337\,m/s^2$$

$\boxed{\Delta g = 33.7\,mm/s^2}$ SMALLER THAN $52\,mm/s^2$

BUT EARTH ISN'T SPHERICAL! ($M_{EARTH} = 5.97 \times 10^{24}\,kg$)

$$R_{POLE} = 6357\,km, \quad R_{EQ} = 6378\,km$$

$$\Delta g = \frac{GM}{R_p^2} - \frac{GM}{R_E^2} + \omega^2 R_E = (6.67 \times 10^{-11})(5.97 \times 10^{24})\left(\frac{1}{(6357 \times 10^3)^2} - \frac{1}{(6378 \times 10^3)^2}\right) - \omega^2 (6378 \times 10^3)$$

$$= 0.0985\,m/s^2 = \boxed{98.5\,mm/s^2}$$

ARG! TOO BIG. NOT THE ENTIRE STORY!
NEED MASS DENSITY DISTRIBUTION